Please check that this question paper contains 9 questions and 02 printed pages within first ten minutes.

[Total No. of Questions: 09] [Total No. of Pages: 02]

Uni. Roll No. ....

Program: B.Tech. (Batch 2018 onward)
Semester: 1/2
Name of Subject: Mathematics I
Subject Code: BSC-103
Paper ID: 15927
Scientific calculator is Not Allowed

MORNING
10 MAY 2023

Time Allowed: 03 Hours **Max. Marks: 60**

NOTE:

1) Parts A and B are compulsory
2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
3) Any missing data may be assumed appropriately

Part – A **[Marks: 02 each]**

Q1

a) State necessary and sufficient condition for the differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$ to be exact.

b) Evaluate the improper integral $\int_0^{\pi/2} \tan x\, dx$.

c) State Cauchy Integral test.

d) Evaluate $\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}$.

e) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$.

f) Using Cayley Hamilton Theorem, Find the inverse of $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.

Part – B **[Marks: 04 each]**

Q2. State and prove necessary condition for convergence of a positive term series.

Q3. Find the general solution of the differential equation
$(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$.

Q4. Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ using Taylor's Theorem.

P.T.O.

Q5. Prove that $\beta(m,n)=\frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$.

Q6. Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ by variation of parameter method.

Q7. For what value of k, the equations $x+y+z=1$, $2x+y+4z=k$, $4x+y+10z=k^2$ have a solution and solve them completely in each case.

Part – C **[Marks: 12 each(06 for each subpart if any)]**

Q8. Solve $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-20y=(x+1)^2$.

OR

(i) Solve $e^{4x}(p-1)+e^{2y}p^2=0$.

(ii) Solve the differential equation $\frac{dy}{dx}+y=y^2$.

Q9. Diagonalize the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ and obtain the modal matrix.

OR

Discuss the convergence of the series $1+\frac{2x}{2!}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+\frac{5^4x^4}{5!}+\ldots$.

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